

eMATH**Centripetal Motion****Purpose:**

1. To plot a centripetal force-speed graph for an object moving in a circular path.
2. To straighten the graph so that the slope can be determined.
3. To use the slope to determine the mass of the object.

Background:**Newton's first law states:**

An object will continue either being at rest or moving at a constant velocity unless acted upon by an external non-zero net force.

An object moving at a constant velocity will continue moving in the same direction at the same speed unless acted upon by an external net force. For an object to move in a circular path, its direction (i.e., velocity) must be continually changing. Therefore, a net force must be acting on it. The net force is the centripetal force and acts perpendicular to the velocity vector. The magnitude of the velocity is the object's speed and its relationship to the centripetal force is expressed as:

$$F_c = \frac{mv^2}{r}$$

The purpose of this eMATH is to plot the centripetal force as a function of speed and to analyze the graph to determine the mass of the object.

Curve Straightening:

A centripetal force vs. speed graph produces a parabolic curve. In this exercise the curve must be turned into a straight line so that its slope can be determined and the mass of the object can be found.

The reason that a centripetal force vs. speed graph is not linear is that force is a function of the square of the speed. This forms a quadratic relation, and a parabolic-shaped line.

To straighten this line, we must make the equation for centripetal force a linear one of the form $y = mx + b$. In essence, the v^2 must be replaced with a non-squared value. To do this we can make a variable " u " equal to v^2 . Writing this mathematically, we say:

$$\text{Let } u = v^2$$

Rewriting the equation for centripetal force using " u " produces a linear relationship:

$F_c = \frac{mv^2}{r}$, which can be expressed as $F_c = \left(\frac{m}{r}\right)v^2$, so that it more closely resembles the straight-line relationship of the form $y = mx + b$. If we write both formulas, one overtop the other, it is easier to see how the two equations are similar and how the variables match up:

$$F_c = \frac{m}{r}v^2$$

$$y = mx + b$$

F_c represents the "y" values, $\frac{m}{r}$ is the slope, and v^2 represents the "x" values. In this case, the "b" value (y-intercept) equals zero.

A new data table will need to be created that will be used to plot centripetal force as a function of v^2 .

Instructions:

1. Download and open the accompanying Excel spreadsheet. Use an appropriate application or graphing calculator to plot the values of the centripetal force as a function of speed using the values in the table. The graph that you plot should be parabolic.
2. The second table represents the centripetal force as a function of " v^2 ." Now fill in the values of " v^2 " in table 2. To do this, keep the values of centripetal force from table 1, but square each value of v and place it the " v^2 " column. Create a graph from the data in table 2 using an appropriate application or graphing calculator. This new graph should be linear.

Analysis:

1. The second graph that plots centripetal force vs. v^2 is a straight line. Determine the slope of the line in the second graph.
2. In this case the slope value represents $\frac{m}{r}$ (see the "Curve Straightening" notes above). Using the value of the slope determined and the radius of curvature of the object moving in the circular path (from the Excel spreadsheet), determine the mass of the object.

Example:

$$\frac{m}{r} = \text{slope}$$

$$m = (\text{slope})r$$

$$m = (\text{slope}) \times 5.00 \text{ m}$$